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ENERGY DISSIPATION IN A SOLID DISPERSION SYSTEM OF SINGLE-DOMAIN FERROMAGNETIC PARTICLES UNDER THE ACTION OF A VARIABLE LINEARLY POLARIZED FIELD

B. É. Kashevskii and I. V. Prokhorov

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The heat power released as a result of magnetic hysteresis in a system of small ferromagnetic particles suspended in a solid matrix acted upon by a linearly polarized magnetic field has been determined theoretically and experimentally. The calculations were done with the use of a model of noninteracting, uniformly magnetized particles with uniaxial anisotropy. Acicular particles of gamma iron oxide were used in the experiments. It is shown that the heat power released as a result of magnetic hysteresis may measure several megawatts per cubic meter, which makes it possible to use disperse magnetic systems as volumetric heaters.

Introduction. As far as we know, ferromagnetic materials releasing heat in the process of their magnetization reversal as a result of magnetic hysteresis in them have not yet been considered as possible heat sources. At the same time, they can be used for volumetric heating of dielectric bodies that are difficult to heat by traditional methods. Of interest is the use of small ferromagnetic particles for this purpose. These particles can form heat sources with adaptable shape and size. The size of such heat sources can be as small as the cell size, which is very urgent in view of the existing tendency toward miniaturization of products and development of nanodimensional technologies.

The aim of the present work is theoretical and experimental determination of the heat power released in a system of small ferromagnetic particles suspended in a solid matrix acted upon by a linearly polarized magnetic field.

Theory. Let us consider a system of noninteracting, magneto-hard, ferromagnetic particles (single-particle approximation) suspended in a solid matrix. The particles are assumed to be sufficiently small to provide their homogeneous magnetization (single-domain). It is known that this state arises when the particle size is smaller than the critical size R_* at which the energy of the magnetization inhomogeneity (domain wall) is higher than the energy of the demagnetizing field of a homogeneously magnetized particle [1]. This size is equal to tens of nanometers for spherical particles of various ferromagnetics. Acicular, high-coercivity particles that are used, for example, in the production of magnetic media, can be in the single-domain state even in the case where their size measures several tenths of a micron [2]. We will describe the dynamics of magnetization of a particle acted upon by a variable field with the use of a coherent rotational model. Since the effective, magnetic anisotropy of acicular particles is mainly determined by the anisotropy of their shape, we will assume that the effective anisotropy is single-axis in character and the orientational state of the system of particles is completely characterized by sets of unit vectors $\mathbf{e}$ and $\mathbf{n}$ in the direction of magnetic moments and the lowest magnetization axes. It is also assumed that the strength of the linearly polarized magnetic field $\mathbf{H}(t) = H(t)\mathbf{h}$, where $\mathbf{h}$ is a fixed unit vector in the polarization direction; $H(t) = H_0 \cos(\omega t)$.

The orientation energy of a particle acted upon by a magnetic field is

$$U = -IVH(\mathbf{eh}) - KV(\mathbf{en})^2 . \quad (1)$$

At a constant external field and a definite orientation of the particle, the position of the magnetic moment is equilibrium if $\partial U/\partial\varphi = 0$, where $\partial\varphi = \mathbf{e} \times \partial\mathbf{e}$ is the vector of magnetic moment rotation. The equilibrium equation usually has the form $\mathbf{e} \times \mathbf{H}_{\text{eff}} = 0$, and the effective strength of the field is defined as

$$\mathbf{H}_{\text{eff}} = -\partial U/\partial\mathbf{m} = \mathbf{H} + H_{\text{a}}(\mathbf{en})\,\mathbf{n} , \quad H_{\text{a}} = 2K/I . \quad (2)$$

A. V. Luikov Heat and Mass Transfer Institute, National Academy of Sciences of Belarus, 15 P. Brovka Str., Minsk, 220072, Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 77, No. 5, pp. 35–40, September–October, 2004. Original article submitted November 18, 2003.

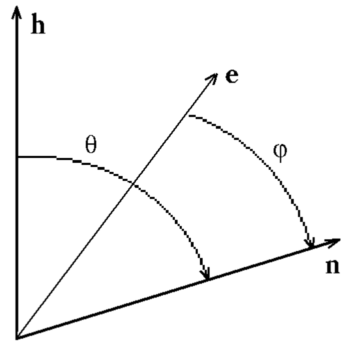

Fig. 1. Scheme of the problem.

Note that the magnetic moment at equilibrium lies in the plane formed by the vectors **H** and **n**. The fulfillment of the necessary condition $\mathbf{e}[\mathbf{n} \times \mathbf{H}] = 0$ is evident in view of the relation $[\mathbf{n} \times \mathbf{H}] = [\mathbf{n} \times \mathbf{H}_{\rm eff}]$ following from (2) and the identity $\mathbf{e}[\mathbf{n} \times \mathbf{H}_{\rm eff}] \equiv -\mathbf{n}[\mathbf{e} \times \mathbf{H}_{\rm eff}]$.

Let us introduce a polar coordinate system with an axis directed along the field of polarization **h**. The orientation state of the system will be determined by the angle of deviation of the anisotropy axis from the polarization direction θ and the angle of deviation of the magnetic moment from the anisotropy axis φ (Fig. 1). Expression (1) for the energy of the particle will have the form

$$u = U/KV = -\,a \cos(\theta - \varphi) - \cos^2(\varphi)\,, \quad a\,(t) = a_0 \cos \omega t\,, \quad a_0 = IH_0/K \equiv 2H_0/(H_{\rm a})\,. \tag{3}$$

The equilibrium position of the magnetic moment at a definite orientation of the anisotropy axis is determined from the equation

$$a \sin\,(\theta - \varphi) = \sin 2\varphi\,, \tag{4}$$

following from the condition $\partial u/\partial\varphi = 0$. Among the solutions of (4), the solutions satisfying the condition $\partial^2 u/\partial\varphi^2 > 0$ are stable, i.e.,

$$a \cos(\theta - \varphi) + 2\cos 2\varphi > 0\,. \tag{5}$$

The behavior of the roots of Eq. (4) is well understood [1]. In the zero field ($a = 0$) it has four roots: $\varphi = 0, \pm\pi/2, \pi$. The extreme values of φ correspond to the minimum energy and the stable magnetic moments oriented in the two physically indistinguishable opposite directions of the easy axis, and the values of $\varphi = \pm\pi/2$ correspond to the unstable magnetic moments oriented transversely to the easy axis, i.e., positioned at the tops of the energy barrier separating the stable states. An external magnetic field applied along the anisotropy axis ($\theta = 0$) increases the potential-well depth in the direction coincident with the field direction ($u(\varphi = 0) = -a - 1$) and decreases the potential-well depth in the opposite direction ($u(\varphi = \pi) = a - 1$). The equilibrium equation has solutions $\varphi = 0$, $\pm\arccos(-a/2)$, and π. As follows from (5), the solution $\varphi = 0$ is stable at all the values of a and the solution $\varphi = \pi$ is stable at $a < 2$ and unstable in stronger fields. Intermediate solutions correspond to the energy maxima.

Thus, in the case where a magnetic field of subcritical value $|a| < 2$ is applied along the anisotropy axis, the magnetic moment can be oriented in one of two directions: along the field (thermodynamically stable orientation) and in the opposite direction (metastable orientation). In supercritical fields $|a| > 2$, the magnetic moment can be oriented only in one direction.

In the case where a variable field with an amplitude a_0 exceeding the critical value $|a_0| > 2$ is applied along the axis of a particle, the magnetic moment changes its direction two times during the period. As a result of each of these changes, the potential energy stored by the particle decreases by $\Delta u = 4$ ($\Delta U = 4KV$). The magnetic moment is transformed into a new stable state through Larmor precession, decaying during the characteristic spin-lattice relaxation time ($\sim 10^{-9}$ sec) because of the "magnetic viscosity" of the particle. For this time, the potential magnetic energy ΔU is converted into the internal (heat) energy of the particle. If the characteristic time of change in the field is large as

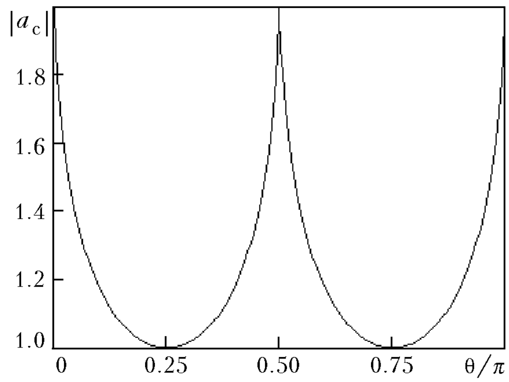

Fig. 2. Modulus of the dimensionless critical field of spasmodic magnetization reversal of a particle as a function of the angle of orientation of its anisotropy axis.

compared to the spin-lattice relaxation time scale, the magnetization reversal of the particle proceeds spasmodically against the background of the change in the field.

Let us consider the case where a magnetic field is applied in the direction perpendicular to the anisotropy axis ($\theta = \pi/2$). The equilibrium equation has solutions $\varphi = \arcsin(a/2)$ and $\pm\pi/2 - \arcsin(a/2)$. The extreme roots of this equation are stable in fields with $|a| < 2$. The root $\varphi = \pi/2$ (the magnetic moment is directed along the field) is unstable in fields with $|a| < 2$ and stable in stronger fields; the direction $\varphi = -\pi/2$ opposite to the direction of the field is always unstable. Thus, in the case considered, in a field with $|a| < 2$ the magnetic moment can be in one of two metastable states (positioned between the field and the anisotropy axis variously directed) which, when the strength $|a| = 2$ is attained, merge in the direction of the field and form a single thermodynamically stable state. The magnetic moment is transformed with increase in the field strength from the metastable to the stable state monotonically in a reversible way.

In the case where the anisotropy axis is arbitrarily directed, the critical field strength at which the system is transformed from the state with two possible orientations of the magnetic moment into a state with a single stable orientation is determined from (4) and the equation

$$a\cos(\theta - \varphi) + 2\cos 2\varphi = 0\,, \tag{6}$$

representing the condition on which the curve $u(\varphi)$ bends as a result of the merge of one of the maxima with one of the minima ($\partial^2 u/\partial\varphi^2 = 0$). Eliminating the angle φ from (4) and (6), we obtain an expression for the critical field:

$$a_c^2 = 4\left[\sin^{2/3}\theta + \cos^{2/3}\theta\right]^{-3}. \tag{7}$$

The graph of $|a_c(\theta)|$ is presented in Fig. 2. As is seen, a critical, spasmodic magnetization reversal field has a minimum of $|a_c| = 1$ at the anisotropy axis orientations $\theta = \pi/4$ and $3\pi/4$ and a maximum of $|a_c| = 2$ at $\theta = 0$, $\pi/2$, and π.

The change in the potential energy of the particle as a result of a rapid change in the field strength (in kilovolts) is defined by the relation

$$\Delta u(\theta) = u(\theta, \varphi_2(\theta), a_c(\theta)) - u(\theta, \varphi_1(\theta), a_c(\theta)) =$$

$$= -2a_c \sin\left(\theta - \frac{\varphi_2 + \varphi_1}{2}\right)\sin\frac{\varphi_2 - \varphi_1}{2} + \cos^2\varphi_1 - \cos^2\varphi_2\,. \tag{8}$$

The value of φ_1 is found by eliminating the quantity a from system (4), (6):

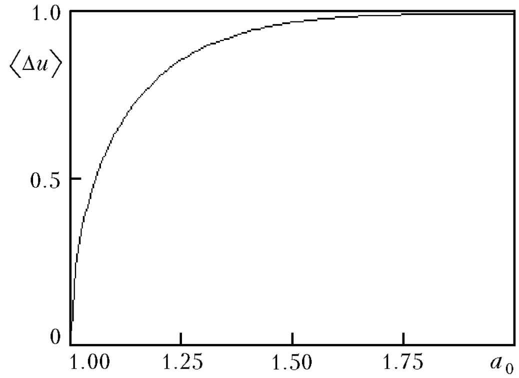

Fig. 3. Relative energy dissipated in an unordered dispersion system of single-domain particles during one cycle of magnetization reversal.

$$\tan^3 \varphi_1 = -\tan\theta \,. \tag{9}$$

The finite position of the magnetic moment is determined from the equilibrium equation (4) at values of θ and $a = a_c(\theta)$ determined from (7). We failed to obtain the desired dependence $\varphi_2(\theta)$ in explicit form. To determine its numerical value, we will assume that the angle of orientation of the particle relative to the polarization axis of the field, selected as the positive direction, is acute ($0 \le \theta < \pi/2$). In this case, the magnetic moment is transformed in the field directed oppositely to the polarization axis. Consequently, the critical field strength is

$$a_c(\theta) = -2\left[\sin^{2/3}\theta + \cos^{2/3}\theta\right]^{-3/2} . \tag{10}$$

In this case, $\varphi_2(\theta)$ is determined from the equation

$$a_c(\theta)\sin(\theta - \varphi_2) = \sin 2\varphi_2 \,. \tag{11}$$

Equation (11) should be solved at $\pi < \varphi_2 < \varphi_1$ since the magnetic moment is transformed to the sector between the direction of the field and the opposite direction of the easy axis.

If the axes of all the particles are directed along the polarization axis, the heat power released as a result of the dispersion of particles with a numerical concentration n (volume concentration $c = nV$) under the action of the field with a frequency of $\omega = 2\pi/T$ is defined as

$$W_{\parallel} = \frac{2KVn}{T}\,\Delta u(0) = \begin{cases} 0\,, & (a_0 < 2)\,; \\ 4Kc\omega/\pi\,, & (a_0 > 2)\,. \end{cases} \tag{12}$$

Let us assume that the particle axes are distributed in a random way. If $\int_0^{\pi} f(\theta)d\theta = 1$, the function of distribution of particles over the orientation angles has the form $f(\theta) = (1/2)\sin\theta$. In the field of amplitude $a_0 > 2$, all particles participate in the dissipation. If $1 < a_0 < 2$, only those particles whose orientation angles satisfy the condition $\theta_{min} < \theta < \theta_{max}$ participate in the dissipation. The minimum and maximum orientation angles (note that only the orientations in the first quadrant are considered) are determined from the relations

$$a_0^2 - 4\left[\sin^{2/3}\theta_{min} + \cos^{2/3}\theta_{min}\right]^{-3} = 0\,, \quad \theta_{max}(a_0) = \frac{\pi}{2} - \theta_{min}(a_0)\,.$$

In this case, the dissipation power is equal to

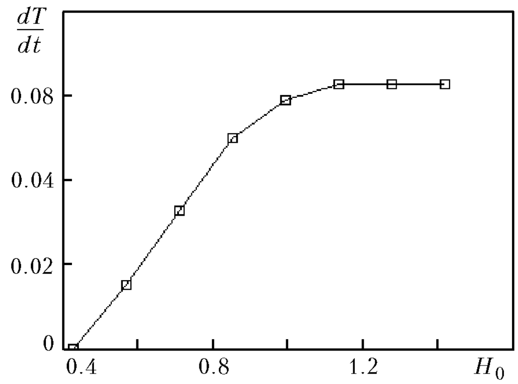

Fig. 4. The dependence of the rate of initial increase in the temperature *dT/dt* of a 10% suspension of gamma iron oxide particles in beeswax on the amplitude of the magnetization reversal field with a frequency of 50 Hz. *dT/dt*, K/sec; H_0, kOe.

$$W(a_0) = \frac{2KVn}{T}\langle \Delta u\rangle = \frac{cK\omega}{\pi}\langle \Delta u\rangle\ , \quad \langle \Delta u\rangle = \begin{cases} 0\ , & a_0 < 1\ ; \\ 2\int\limits_{\theta_{\min}}^{\theta_{\max}} \Delta u(\theta)\, f(\theta)\, d\theta\ , & 1 \le a_0 \le 2\ ; \\ 1\ , & a_0 > 2\ . \end{cases} \tag{13}$$

Multiplier 2 in the expression for W accounts for the double transformation of the magnetic moment of each particle during the period of the field. The dependence of the average energy $\langle \Delta u\rangle$, dissipated by one particle as a result of one spasmodic change in the magnetic moment, on the field amplitude is shown in Fig. 3. Note that this ultimate ($a_0 > 2$) dissipation power of a disordered system measures a fourth of the corresponding power of the completely ordered system.

Experimental. The heat power released was measured at the initial stage of heating of a cylindrical container containing a 10% (volume concentration) suspension of gamma iron oxide particles in beeswax. The container had a length of 3 cm and a diameter of 1.5 cm. The temperature was measured at the center of the container by a thermocouple positioned along the container axis. The signal from the thermocouple was amplified, fed to an analog-to-digital converter, and recorded by a computer. Measurements were carried out with an interval of 0.1 sec during the first 10 sec after a variable, transverse magnetic field of frequency 50 Hz was applied to the cylinder. The field strength was measured by an F43-56 millitesla meter. The measured dependence of the rate of increase in the temperature of the suspension on the field amplitude is presented in Fig. 4.

Discussion. Since the boundaries of the volume studied with a homogeneously distributed heat source have no influence on the temperature at its center at the first stage of heating, the rate of heating *dT/dt* is related to the source power W by the relation

$$W = [c_p^{\mathrm{p}}\rho^{\mathrm{p}} c + c_p^{\mathrm{w}}\rho^{\mathrm{w}}(1 - c)]\, dT/dt\ .$$

At ρ^{w} = 960 kg/m^3, ρ^{p} = 4.8·10^3 kg/m^3, c_p^{w} = 2.9·10^3 J/(kg·K), and c_p^{p} = 0.65·10^3 J/(kg·K), the maximum dissipation power (dT/dt = 0.063 K/sec) is equal to 1.75·10^5 W/m^3.

Let us compare this result with the result of the calculation: $W_{\max} = cK\omega/\pi$. Substituting c = 0.1, K = 1.7·10^4 J/m^3, and ω = 314 sec^{-1} into the above expression, we obtain a value of $W_{\max}$ = 1.75·10^5 W/m^3, which is close to the measured one. As is seen, the measured and calculated dependences of the dissipation power on the field strength are, on the whole, close in character; however, the heat-source intensity increased more smoothly with increase in the field strength in the experiment.

CONCLUSIONS

Our calculations and measurements have shown that a hysteresis heat source has a relatively high power. When the frequency of the field applied increases by an order of magnitude (to 500 Hz), the heat power released also increases by an order of magnitude. For the system considered, the heat power released is equal to 1.75 MW/m^3. The orientation of particles and the use of particles with a high anisotropy constant make it possible to additionally increase the heat power released. Note that we did not consider problems of efficient conversion of the energy of a variable-field generator into the heat energy of a hysteresis heater.

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NOTATION

$a = IH/K$, dimensionless strength of the magnetic field; c, volume concentration of particles; c_p, specific heat at a constant pressure, J/(kg·K); **e**, unit vector in the magnetic moment direction; $f(\theta)$, function of distribution of particles over orientations; **H**, magnetic field strength, Oe; **h**, unit vector in the field-polarization direction; H_a, strength of the magnetic anisotropy field, Oe; $\mathbf{H}_{eff}$, effective magnetic field, Oe; I, magnetization, G; K, magnetic anisotropy constant, Erg/cm^3; $\mathbf{m} = IV\mathbf{e}$, magnetic moment of a particle, $G{\cdot}cm^3$; **n**, unit vector in the anisotropy-axis direction; n, numerical concentration of particles; R_*, critical size of a single-domain particle, m; t, time, sec; T, period of change in the field, sec; V, volume, cm^3; U, orientation energy of a particle, Oe; $u = U/KV$, dimensionless orientation energy of a particle; W, source power, W/m^3; ΔU, change in the energy of a particle as a result of spasmodic magnetization reversal, Oe; $\Delta u = \Delta U/KV$, dimensionless change in the energy of a particle as a result of the spasmodic magnetization reversal; φ, angle of deviation of the magnetic moment from the anisotropy axis; φ_1 and φ_2, angles of equilibrium orientation of the magnetic moment before spasmodic change and after it; θ, angle of deviation of the anisotropy axis from the polarization direction; ρ, density, kg/m^3; ω, frequency, sec^{-1}. Subscripts: 0, amplitude value; max, maximum value; min, minimum value; $\parallel$, parallel; p, particle; w, wax; a, anisotropy, c, critical; eff, effective; p, pressure.

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